William Tyndale Primary School

Calculation Framework

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INTENT

Our goal for Maths education is that children are able to solve increasingly complex routine and non-routine problems by developing:

- deep, secure and adaptable conceptual understanding;
- fluency with mathematical fundamentals and procedures; and
- proficiency with reasoning, application and use of mathematical vocabulary.

IMPLEMENTATION

Our approach

At William Tyndale, we have adopted a teaching for Mastery approach in order to deepen the understanding and improve progress for all children. Pupils are encouraged to develop a broad and deep understanding of maths in order to provide an excellent foundation for all future maths learning. We work closely with our Maths Hub and the National Centre for the Excellence in the Teaching of Mathematics (NCETM) to develop our pedagogy, which focuses on using the 5 Big Ideas of Mastery: fluency, coherence, variation, mathematical thinking and structure/representation.

Fluency

All classes from Year 1-6 spend time each day practising their factual fluency; this may include Big Maths, Beat That, Doodle Maths or other fluency practice. These short but focused sessions may involve practising number facts or times tables. This regular practice is supporting children to have greater confidence with the fluency that is required to support the learning in their maths lessons.

Maths Meetings

Years 1-3 hold regular sessions outside of the maths lesson with a focus on time, measure, shape, data handling and number. By recapping these topics regularly, it ensures that the concepts are secure in children's long-term memory.

Coherence

Maths lessons build on the prior learning in order to ensure that all children can remember the necessary points in order to successfully retain the new information. Lessons are planned with small steps in order to support children to understand new concepts gradually in order that they are confident and secure. Sequences of lessons are planned carefully to ensure that links are made between areas of learning. We use the White Rose small steps, NCETM Spine materials, and Maths No Problem among other resources.

Variation

At William Tyndale, we believe that children are more able to learn and retain new concepts when they are represented in a variety of ways to show what they are and what they are not. Attention is paid to the selection and order of examples with the aim of avoiding mechanical repetition and developing intelligent practice.

Mathematical Thinking

Lessons involve opportunities to make links between areas of maths. Fluency is put into practice and pupils are encouraged to use their logical thinking and reasoning skills regularly. We encourage the use of talk partners for children to discuss and further their thinking. Explanation of a concept is a demonstration of understanding. Stem sentences are used to support the articulation of their understanding. Children are given opportunities to view concepts in different ways to make links and to deepen their thinking about maths.

Representation

Attention is paid to the structures of the maths and how best to represent them. We use a wide range of physical and pictorial resources to support children to understand the structure of maths, so that they are able to move towards the abstract understanding.

FOUNDATIONS FOR CALCULATION

Place Value

As a foundation for learning strategies for addition, subtraction, multiplication and division, children first develop a strong understanding of the concept of number and counting, through exploration of place value.

They are also introduced to the concept of parts and wholes and learn that a number can be partitioned into two or more parts. They are introduced to the equals sign, and the addition, subtraction, multiplication and division symbols. They also develop their understanding of equality and inequality. These concepts are revised through the study of place value at the start of each year.

As a part of place value learning in KS2, children explore the whole place value chart (100 million to thousandths) in groups of three, which encourages relational understanding. They develop strong conceptual understanding of the value of each place, as well as of the importance of place holders. As a part of this, they learn to multiply and divide by 10, 100 and 1000, and practise this regularly to develop fluency.

millions			thousands			ones			decimals and fractions		
100,000,000	10,000,000	1,000,000	100,000	10,000	1000	100	10	1	0.1 $\frac{1}{10}$	0.01 $\frac{1}{100}$	0.001 $\frac{1}{1000}$

Times Tables

Fluency with times tables is vital for children to be able to perform complex calculations using formal written algorithms. From Reception to Year 4, children learn the conceptual foundations and the specific times tables outlined below. They learn to make links between the times tables and to find related facts. This secure foundation of understanding ensures that children are proficient with mentally recalling times tables facts when applying them to calculations in Years 5 and 6. The progression of learning is outlined below.

RECEPTION	Children learn to double numbers up to 10, using concrete materials. They identify what 'double' and 'not double' are, as a part of their developing understanding of equality and inequality. They explore this as a part of learning to subitise up to 6.
YEAR 1	Children learn to skip count in 2s, 5s, and 10s. As with all counting, they learn to count both forwards and backwards, so that they are able to apply this to all four operations in future year groups.
YEAR 2	Children continue to practise skip counting 2s, 5s and 10s, as well as being introduced to counting in 3s. Children are explicitly taught the 2-, 5- and 10- times tables, practising recalling full times table facts (3 x 5 = 15), both in and out of order.
YEAR 3	Children learn the 2-, 4- and 8- times tables as a group, then the 3- and 6-times tables as a group. The grouping of these times tables encourages children to apply their knowledge of doubling, which helps to develop secure and long-lasting understanding. They then learn the 12-times table as a combination of the 10- and 2-times tables, which they are already secure with. They continue to skip count and to practise recalling full times table facts, both in and out of order.
YEAR 4	Children learn the remaining times tables (9, 11 and 7), in which they explore patterns and facts already known from other times tables. They continue to practise all times table facts up to 12x12, both in and out of order, which prepares them for the statutory *Multiplication Tables Check* at the end of Year 4.

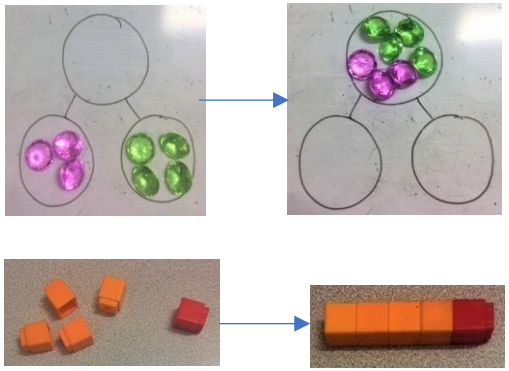
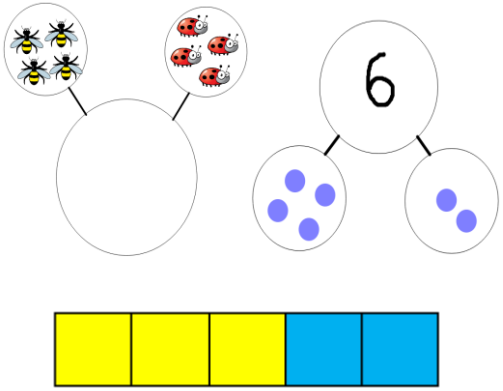
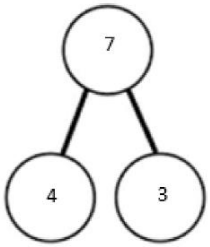
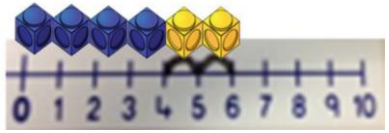
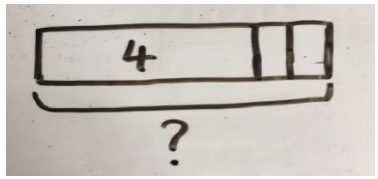
CALCULATION STRATEGIES

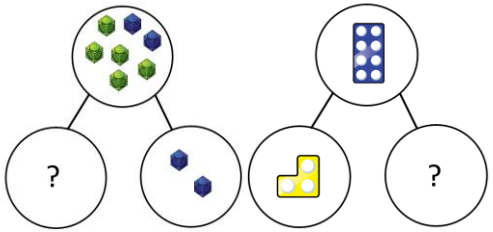
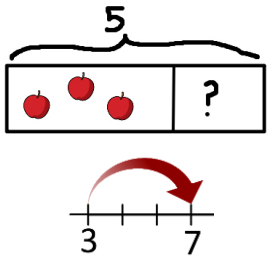
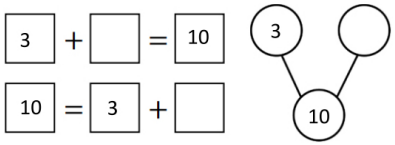
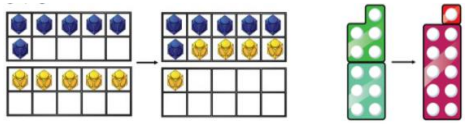
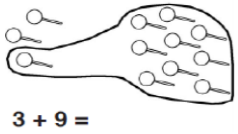
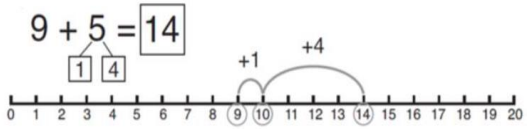
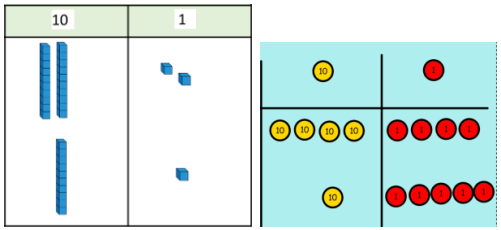
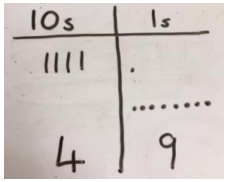
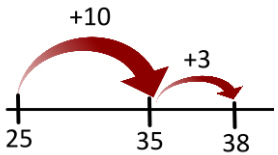
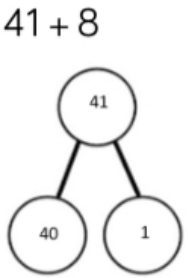
The following pages outline the progression in calculation strategies and key vocabulary for the four operations: addition, subtraction, multiplication and division. Children learn these strategies across year groups from EYFS to Year 6, as appropriate to their stage of development in mathematical understanding. The focus of learning throughout is on developing conceptual understanding through the use of concrete resources and pictorial representations, before moving on to more formal written methods of calculation.

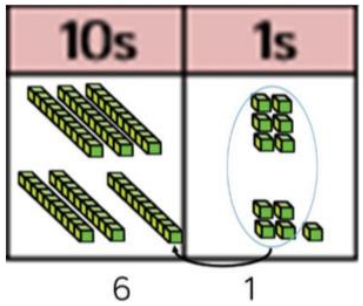
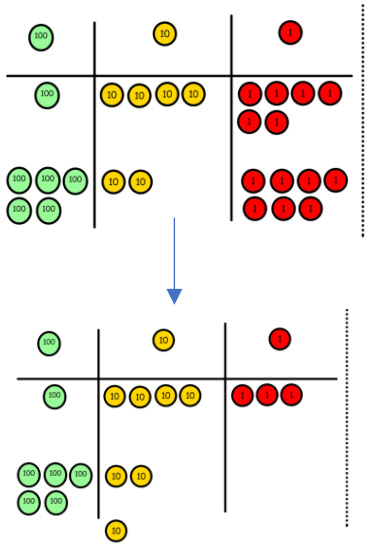
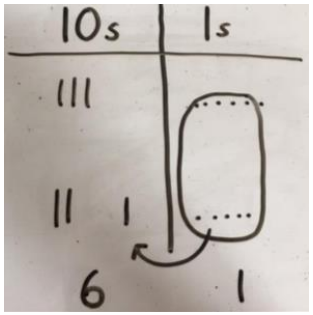
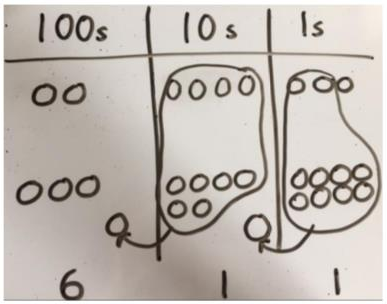
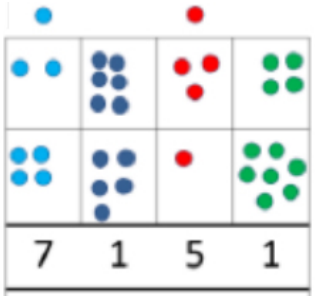
When a new strategy is introduced, the previously learnt strategies are revisited as a part of conceptual variation. This enables children to develop strong conceptual understanding, and to make links between different aspects of mathematics. For example, when learning to add 4-digit numbers in Year 4 using column addition, children will first practise combining parts together, adding more to an existing amount, finding a part of a whole, and regrouping, to ensure they are ready to master the formal written algorithm.

Addition

Key vocabulary	sum, total, whole, parts and wholes, plus, add, altogether, more, 'is equal to', 'is the same as'		**summa –** total (noun) **addere –** to add (verb) **cumulare –** to increase (verb) **totaliter -** altogether (adverb) **totus** – all/whole/total (adjective)

Strategy	Concrete	Pictorial	Abstract
Adding together: combining two parts to make a whole	Children use objects to add two numbers together, using a part-whole model. 	Children use pictures to add to numbers together or they represent objects using dots in a part-whole model or using a bar. 	Children use the objects and models they have explored to move into the abstract.  ***Four add three is equal to seven.*** ***4 + 3 = 7*** ***Seven is equal to three add four.*** ***7 = 3 + 4***
Adding more: adding onto a group	Children add more to a group by counting on. 	Children count on using number lines, building on their use of concrete objects.  The bar model encourages children to count on rather than count all. 	Children put the larger number in their head and count on the smaller number to find the total. **5 + 12 = 17** *"Put twelve in your head, now count on five more."*

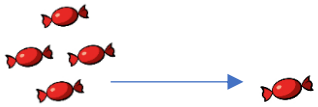
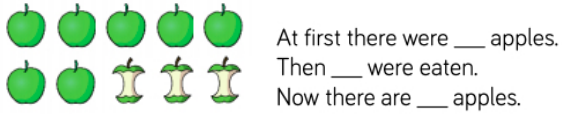
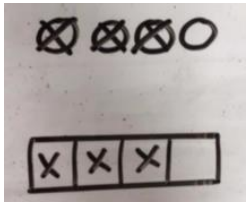
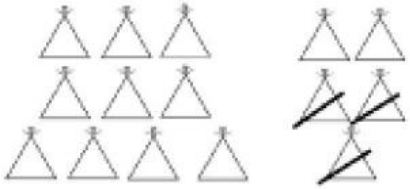
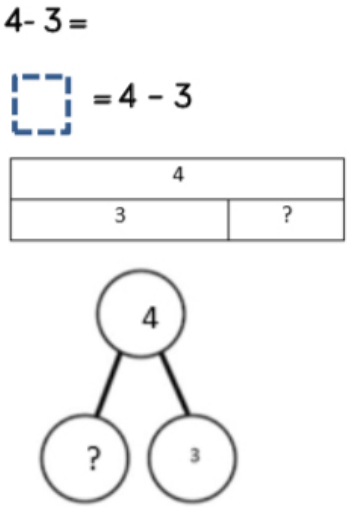
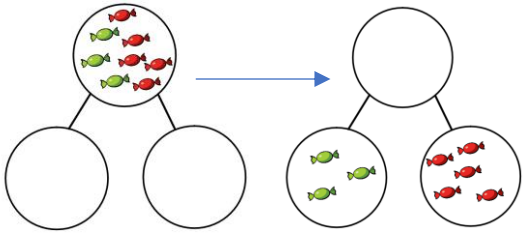
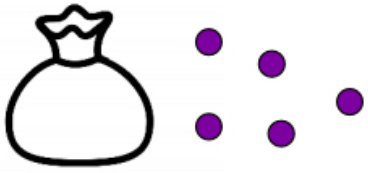
Finding a part (missing number)	Children are given the whole and one part. They count on from the known part to the whole, to find the missing part. 	Children use bar models and number lines to begin to develop the concept of difference, using counting on. 	Children find missing parts to complete missing number problems and fact families.  ***3 is a part,*** ***___ is a part,*** ***10 is the whole.***
Regrouping to make 10 (bridging 10)	Children start with the bigger number and use the smaller number to make ten then see how many more they have. They use tens frames and cubes/counters or Numicon. 	Children use pictures or a number line. Regroup or partition the smaller number to make 10.  3 + 9 = 	Children solve problems in the abstract, mentally making ten then counting on. ***7 + 4 = 11*** ***If I am at seven, how many more do I need to make ten? How many more do I need to add to find the total?***
Column method using base 10 (partitioning, no regrouping)	Children use dienes or place value counters to add numbers together. They start on the right, combining the numbers to find the total. This lays the conceptual foundation for formal column addition. 	Children use sticks and dots to represent base 10. **41 + 8**  They also use partitioning to add using an empty number line. 	Children partition numbers to add them together, starting in the ones.  1 + 8 = 9 40 + 9 = 49

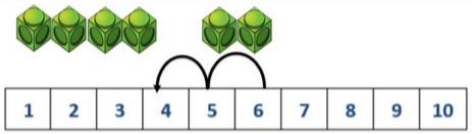
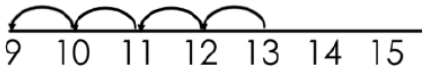
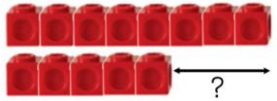
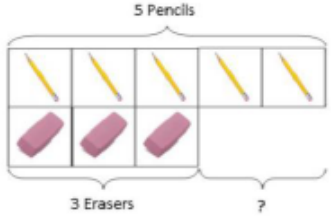
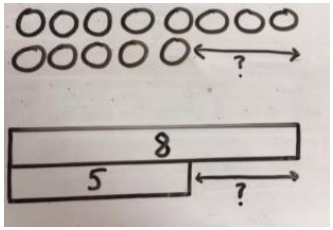
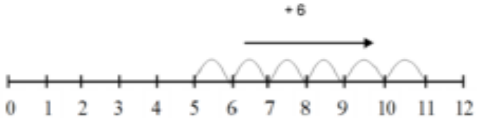
Column method using base 10 (partitioning, regrouping)	Children use dienes or place value counters to make both numbers on a place value grid. They add the numbers in each column together, starting on the right.  When there is more than 9 in a column, they regroup into the next column. This is introduced initially as a 'fair swap' (10 ones is equal to 1 ten).  As children move on to decimals, money and decimal place value counters are used to support learning.	Children draw pictorial representations of the columns and place value counters or dienes to further support their understanding.   As they begin to use representations that lay the foundation for the formal written algorithm, they regroup above the calculation.  	Children use their understanding of partitioning to use an expanded method for addition. This mirrors their use of concrete resources, developing their understanding of the numerical values of the place value counters. **325 + 173** 100 10 1 300 20 5 100 70 3 400 90 8 They then recombine the number to record the total. **400 + 90 + 8 = 498** They practise this with and without regrouping. 10 400 20 5 100 60 6 500 90 11

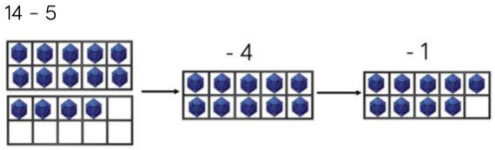
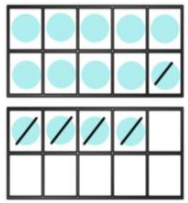
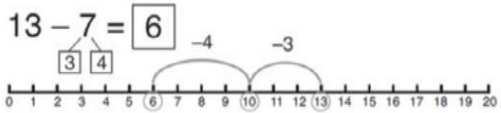
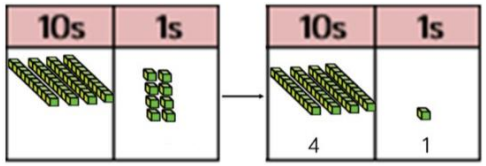
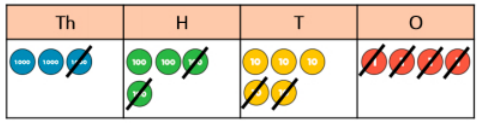
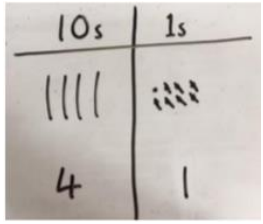
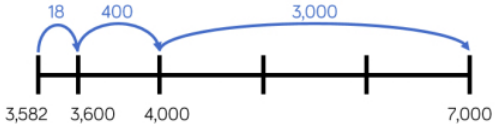
	Year 3	Year 4	Year 5	Year 6
Progression towards a formal written algorithm for addition	Children are introduced to the formal written algorithm for column addition. They learn to turn a number sentence into a column layout by lining the digits up in place value columns with the larger number on top. They start the calculation on the right, regrouping above the calculation when necessary. They say aloud what they are doing at each stage of the calculation to reinforce the learning. **64 + 356** becomes (+1)(+1) 3 5 6 + 6 4 4 2 0 ***"Six ones add four ones equals ten ones. Record 0 as a place holder in the ones column and regroup the ten. Five tens add six tens is equal to eleven tens and one more is twelve. Record 2 in the tens and regroup the hundred. Three hundred and one more is four hundred. The total is 420."***	In Years 4 and 5, children continue to develop confidence with this method using larger numbers and decimals. **4982 + 1142** becomes (+1)(+1) 4 9 8 2 + 1 1 4 2 6 2 2 4 When adding decimals, they line the digits **and** the decimal points up and start on the right. **£35.99 + £21.99** becomes (+1) (+1) 3 5 . 9 9 + 2 1 . 9 9 £ 5 7 . 9 8		In Year 6, children practice the algorithm with numbers up to 100 million and decimals to three decimal places. They move towards the final algorithm below, in preparation for the standard algorithms they will use at secondary school: 1 1 1 1 2 1 7 3 9 6 + 8 1 3 9 0 5 1 0 3 1 3 0 1 (regrouping above) or 2 1 7 3 9 6 + 8 1 3 9 0 5 1 0 3 1 3 0 1 1 1 1 1 (regrouping below)

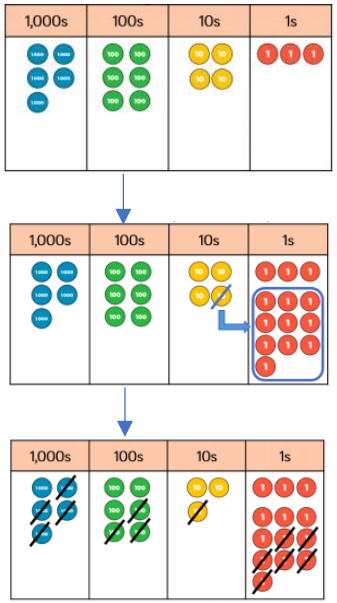
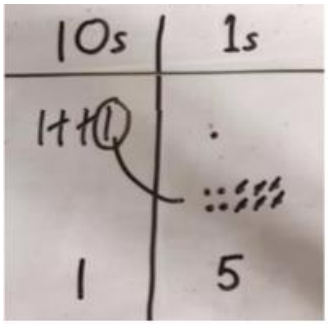
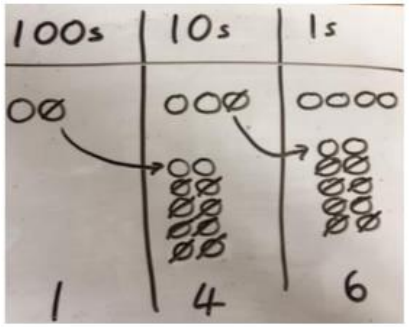
Subtraction

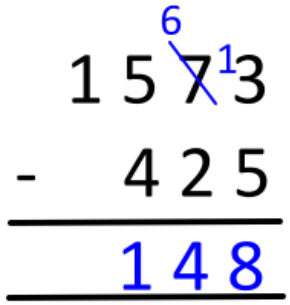
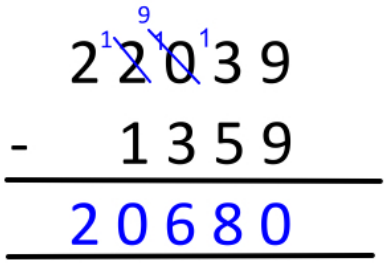
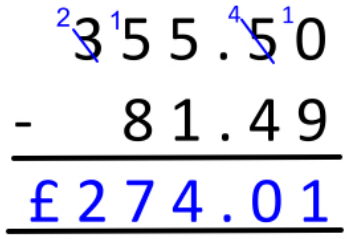
Key vocabulary	take away, less than, the difference, subtract, minus, fewer, decrease		**trahere –** to take (verb) **tractus –** taken (past participle) **sub –** under **subtrahere –** to take away/remove **subtractus –** taken away **decrescere** – to decrease (verb)

Strategy	Concrete	Pictorial	Abstract
Physically taking away (how many are left?)	Children use physical objects such as cubes, counters etc to solve subtraction problems by physically removing the correct number of objects one at a time and counting how many are left. ***I had 4 sweets then I ate 3 of them. How many are left?***  They use number stories to describe the calculation. 	Children cross out drawn objects to show what has been taken away.  	
Breaking apart	Children build on their understanding of finding a part to subtract by partitioning – breaking the whole into two parts. 	Children interpret pictorial representations as subtraction by partitioning. ***In total, there are 8 counters. How many counters are in the bag?*** 	Children use models and images to create number fact families, recording addition **and** subtraction number sentences in the abstract.  __ + __ = __ __ + __ = __ __ – __ = __ __ – __ = __ __ = __ + __ __ = __ + __ __ = __ – __ __ = __ – __

Counting back	Children make the whole and count back, using objects such as cubes or bead strings. They develop understanding that the number they count back **to** is the remainder. This encourages them not to have to re-count the objects remaining. **13 - 4**  They also link this to the number track. 6 – 2 = 4  	Children count back on a number line or number track. They start at the bigger number and count back the smaller number showing the jumps on the number line. **13 - 4**  	Children put the larger number in their heads, then count back the smaller number. **17 – 5 = 12** *"Put seventeen in your head, now count back five more. You can use your fingers to help you count back the correct amount."*
Finding the difference	Children use objects to compare amounts. They are link the concept of difference to finding a missing part.  How many more cakes does Whitney have than Teddy? Whitney  Teddy  Whitney has ___ more cakes than Teddy. They use basic bar models with objects to find the difference.  	Children draw the objects they have used to find the difference or draw simple bar models to compare amounts.  Children use number lines to count on to find the difference. ***What is the difference between 11 and 5?***  	Children work in the abstract, recognising the difference as subtraction and as counting on. They use comparison as a basis for finding numerical patterns. Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.

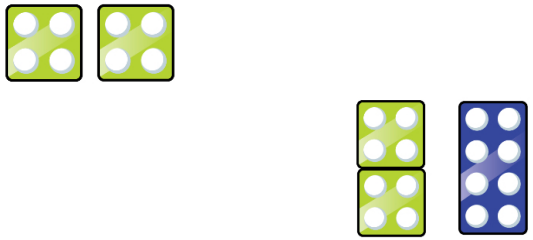
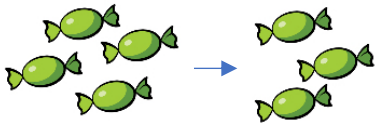
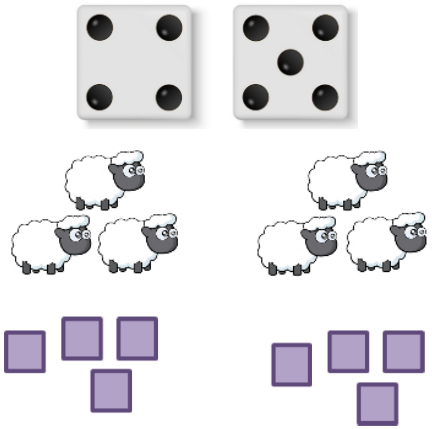
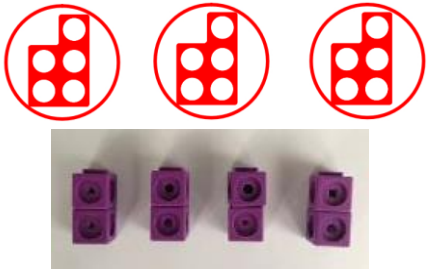
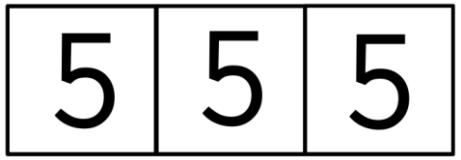
Making 10/regrouping (bridging 10)	Children make the bigger number using tens frames. They subtract to make ten, partitioning the smaller number to find out how many more to take away. 	Children follow the same process by drawing their own representations using tens frames. They also use number lines to bridge ten.  	Children complete this process in the abstract, articulating what they are doing as they solve the problem. **16 – 8=** ***How many do we take away to reach the next 10?*** ***How many do we have left to subtract?***
Partitioning using base 10 (no regrouping)	Children make the bigger number using base 10. They partition the smaller number to subtract, starting on the right. **48 - 7**  **3454 - 1224** 	Children use sticks and dots to represent base 10. **48 - 7**  They also use partitioning to subtract on an empty number line. 	Children partition the smaller number to subtract it from the larger number. **28 – 13** **10 3** **28 – 3 = 25** **25 – 10 = 15**

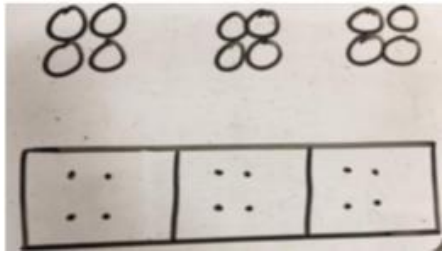
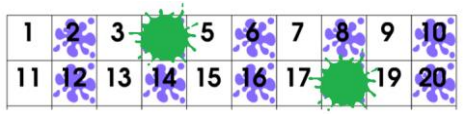
Partitioning using base 10 (regrouping)	Children use base 10 to make the larger number in the place value grid. They subtract the smaller number, starting on the right. If they need to subtract more than value of a column, they regroup from the next column, building on their understanding of a 'fair swap' (10 ones is equal to 1 ten). **5643 - 4316**  As children move on to decimals, money and decimal place value counters are used to support learning.	Children represent base 10 pictorially, showing the exchange.   	Children use their understanding of partitioning to use an expanded method for subtraction. This mirrors their use of concrete resources, developing their understanding of the numerical values of the place value counters. **1573 - 425** 60 1000 | 500 | ~~70~~ | ^{1}3 400 | 20 | 5 1000 | 100 | 40 | 8 They then recombine the number to record the total. **1000 + 1000 + 40 + 8 = 1148** They practise this with and without regrouping.

	Year 3	Year 4	Year 5	Year 6
Column method: Progression towards a formal written algorithm for subtraction	Children are introduced to the expanded method for subtraction (see above), supported by continued use of concrete resources.	Children are introduced to the formal written algorithm for column subtraction. They learn to turn a number sentence into a column layout by lining the digits up in place value columns with the larger number on top. They start the calculation on the right, regrouping from the left when necessary. In Year 4, they start with regrouping in only one column and build up to more complex calculations as appropriate. They say aloud what they are doing at each stage of the calculation. **1573 – 425** becomes  	Children continue to develop confidence with this using this method for more complex calculations and for those involving decimals.  When subtracting decimals, they line the digits **and** the decimal points up and start on the right. **£355.50 - £81.49** becomes  	Children practise and consolidate the formal written algorithm using numbers up to 100 million and decimals up to three decimal places.

Multiplication

Key vocabulary	double, times, multiplied by, the product of, groups of, lots of, equal groups		**multiplicare** - to multiply/to repeat (verb) **duplicare** – to double/to duplicate (verb) **aequare** – to compare/to equal (verb)

Strategy	Concrete	Pictorial	Abstract
Doubling	Children use practical activities to explore 'double' and 'not double'. **double**  **not double** 	Children identify 'double' and 'not double' in pictures. ***Tick the pictures that show double:*** 	Children solve problems related to doubles without concrete or pictorial resources. ***Double four is equal to...*** ***Three two times is equal to...***
Make equal groups	Children make equal groups. They describe how many groups and how many in each group. They focus on value as opposed to objects used.  ***There are ___ groups.*** ***There are ___ In each group.***	Children identify equal groups in pictures, describing how many groups and how many in each group.  They continue to focus on value rather than objects used: 	Children begin to explore equal groups with numbers, continuing to focus on number of groups and number in each group. They do not explore the total at this stage. They explore this using the bar model. 

<table>
<tr>
<td>Counting in multiples (skip counting)</td>
<td>Children use the objects they have put in equal groups to skip count forwards and backwards. They continue to practise this until they know the patterns by rote.
 
2 4 6 8
 
5 10 15 20 25 30</td>
<td>Children draw their own representations to practise skip counting.
  </td>
<td>Children skip count in sequences.
2, 4, 6, 8, 10
Children fill in missing numbers in skip counting patterns.
5, ___, 15, 20, ___, 25
Which numbers are missing from the 2s sequence?
 
</td>
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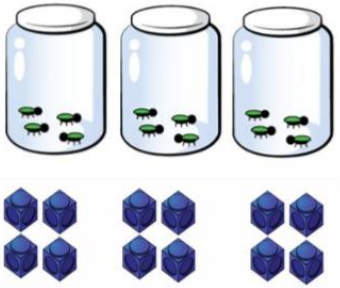
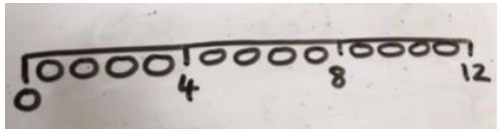
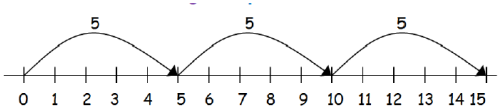
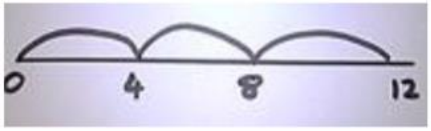
<tr>
<td>Add equal groups (repeated addition and multiplication)</td>
<td>Children skip count equal groups to find the total. They articulate this as repeated addition and as multiplication.
 
There are 3 groups.
There are 4 in each group.
4 + 4 + 4 is equal to 12.
3 groups of 4 is equal to 12.</td>
<td>Children use number lines to show repeated groups. They continue to develop understanding that the final number they skip count to is the total.
 

5 + 5 + 5 = 15
3 x 5 = 15</td>
<td>Children use more abstract representations of repeated addition articulate these as both addition and multiplication sentences.
 
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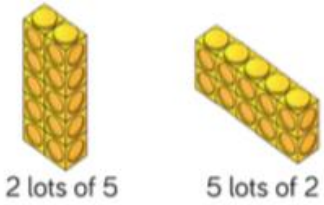
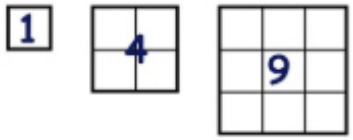
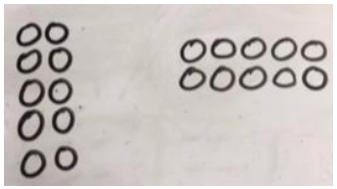
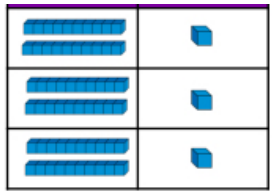
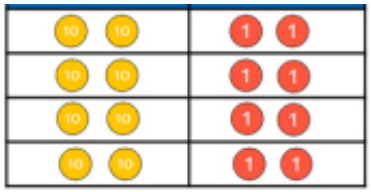
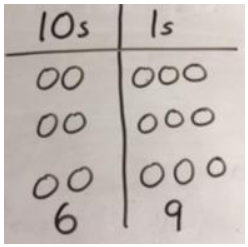
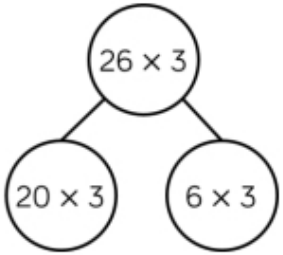
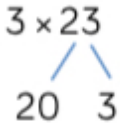
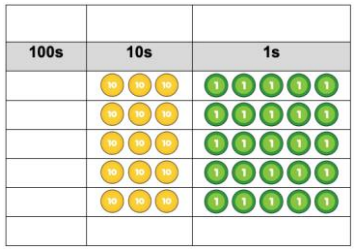
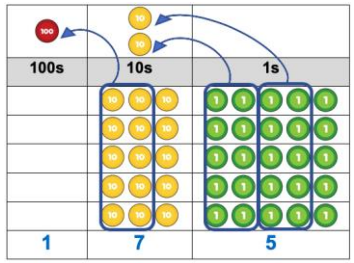
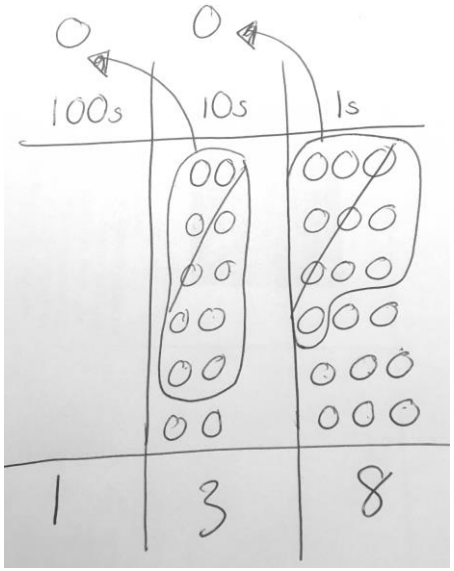
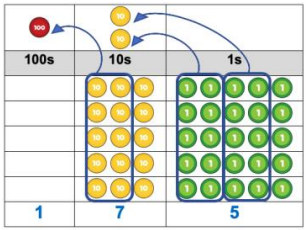
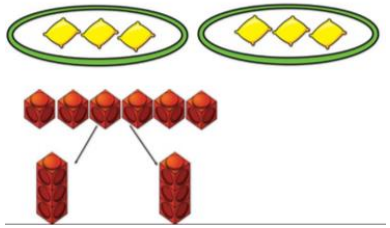
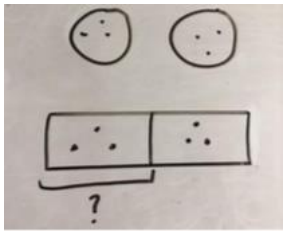
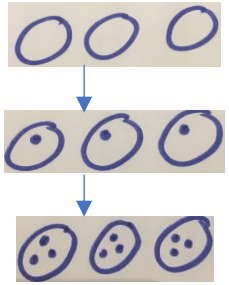
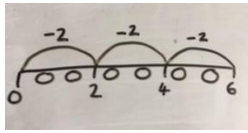
Arrays to show commutativity	Children explore arrays to develop understanding of the commutativity of multiplication.  **2 x 5 is equal to 5 x 2**	Children look at representations of square numbers and draw their own representations to explore commutativity.  	Children use arrays to write a range of calculations. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
Partitioning to multiply (distributive law, no regrouping)	Children use partitioning to multiply two-digit numbers. They lay the numbers out in a column, then start adding the columns on the right to find the total. **21 x 3**  **22 x 4**  As children move on to decimals, money and decimal place value counters are used to support learning.	Children draw or interpret pictorial representations to partition numbers to multiply using the distributive law. **3 x 23**   	Children work in the abstract to partition numbers and multiply using the distributive law.  ***3 x 3 = 9*** ***20 x 3 = 60*** ***60 + 9 = 69***

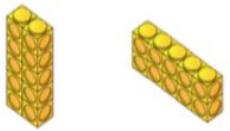
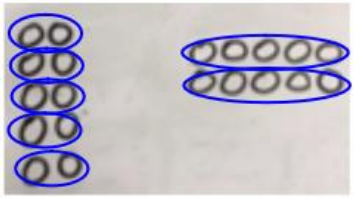
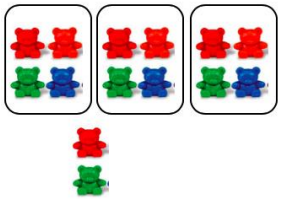
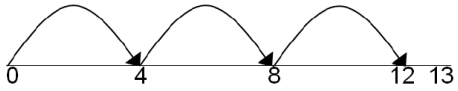
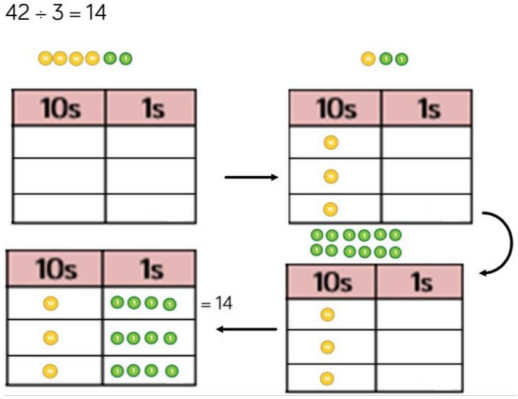
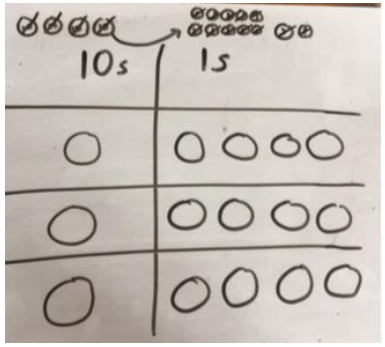
Partitioning to multiply (distributive law, regrouping)	Children use base 10 to make the number the correct number of times in the place value grid. **35 x 5 (thirty-five, five times)**  They find the total in each column, starting on the right. If there are more than 9 in a column, they regroup above the next column (to the left). They then record the total, making sure to add the regroupings.  Children practise this with numbers appropriate to the year group, including decimals. **22.2 x 6 (22.2 six times)**  	Children complete the same process by drawing the representations of base 10.  	**See below**

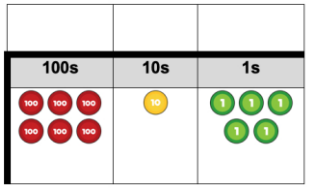
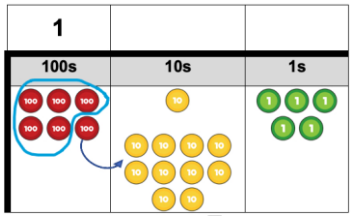
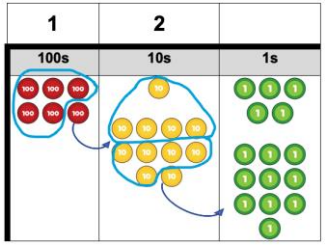
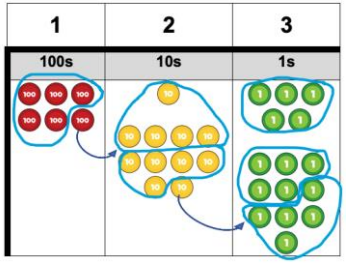
	Year 3	Year 4	Year 5	Year 6
Progression towards a formal written algorithm for multiplication	Children being to record number sentences for the calculations they complete using concrete materials. They do not lay these out as a column method. **35 x 5**  **5 x 5 = 25** **30 x 5 = 150** **150 + 25 = 175**	Children are introduced to the formal written algorithm for multiplying 2-, 3- and 4-digit numbers by a 1-digit number. They learn to turn a number sentence into a column calculation by putting the larger number at the top. **327 x 5** They start multiplying on the right, recording their calculations in a 'brain splat' next to the column. (+1)(+3) 3 2 7 x 5 1 6 3 5 35 10 + 3 15 + 1 Children say aloud what they are doing as they do it. ***"5 times 7 is 35. Record the 5 and regroup the 3 tens above. 5 times 2 is 10, plus the 3 is 13. Record the 3 and regroup the 1 above. 5 times 3 is 15, plus the 1 is 16."***	Children learn to multiply 2-, 3- and 4-digit numbers by a 2-digit number. They start on the right, recording their working in a 'brain splat', and the regrouping above. (+1)(+3) 3 2 7 x 2 5 1 6 3 5 35 10 + 3 15 + 1 When they have finished multiplying by the ones, they cross out all the working for that step. They start multiplying in the tens, first put a place holder (0) into the ones column. (+1) (+1)(+3) 3 2 7 x 2 5 1 6 3 5 6 5 4 0 35 10 + 3 15 + 1 14 4 + 1 6 When they have finished multiplying by the tens, they cross out the working for that step and start the addition calculation to find the overall total. (+1) (+1)(+3) 3 2 7 x 2 5 (+1) 1 6 3 5 + 6 5 4 0 8 1 7 5 35 10 + 3 15 + 1 14 4 + 1 6	Children learn to multiply decimals by an integer. They learn to put the decimal on top and the integer in the **farthest right column** (not in the ones). They then multiply as before. (+1) (+2) 6 . 2 5 x 5 3 1 . 2 5 25 10 + 2 30 + 1 Children also continue to practise multiplying by 2-digit numbers. When they are procedurally fluent, they move towards the final algorithm below: 1 1 3 1 3 2 7 x 2 5 6 6 3 5 2 6 5 4 0 3 3 1 7 5 1 1

Division

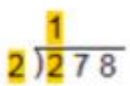
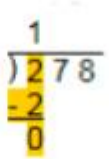
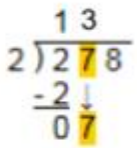
Key vocabulary	share, group, divide, divided by, half, equal parts, equal groups		**dividere –** to divide (verb) **par –** equal/a match for (adjective) **aequare** – to compare/to equal (verb)

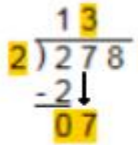
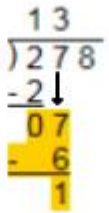
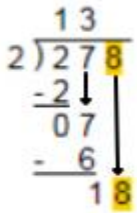
Strategy	Concrete	Pictorial	Abstract
Sharing (halves)	Children learn to share the whole into two equal parts by sharing objects one at a time. 	Children represent this pictorially by drawing groups and sharing the dots one at a time between them. 	Children use their knowledge of doubles and the two times table to halve in the abstract, making links to the inverse. ***2 lots of 3 is equal to 6 so 6 shared into two equal parts is 3.*** 
Sharing (equal groups)	Children share objects into equal groups, one at a time, then they describe the groups they have made.  ***There are 4 groups.*** ***There are 2 in each group.*** ***8 shared into 4 groups equals 2 in each group.***	Children draw the groups, then share the whole counting one by one. **9 ÷ 3** 	Children use times table facts to solve division calculations. ***I know that 6 x 3 is equal to 18, so 18 shared into 3 groups is equal to 6.***
Grouping (repeated subtraction)	Children group the whole into groups of a given number, then count how many groups they have made.  ***12 divided into groups of 4 equals 3 groups.***	Children group the whole by drawing around groups of a given number, or on a number line using repeated subtraction  	Children use times table facts to solve division calculations. They can also use repeated subtraction mentally. **10 ÷ 5** **10 – 5 – 5 = 0**

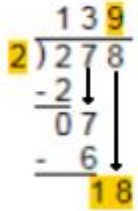
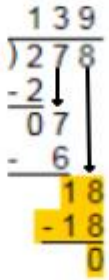
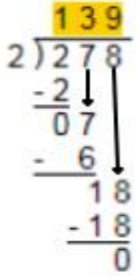
Arrays (link to multiplication)	Children explore arrays to develop understanding of link between multiplication and division, and the difference between grouping and sharing.  ***There are 10 cubes altogether.*** ***10 divided into groups of 2 equals 5 groups.*** ***10 divided into 2 groups equals 5 in each group.***	Children draw their own arrays, grouping the whole differently to build on their understanding of division, and the link with multiplication.  ***10 divided by 5 equals 2*** ***10 divided by 2 equals 5***	Children use arrays to write a range of calculations, including inverse multiplication facts **10 ÷ 5 = 2** **10 ÷ 2 = 5** **2 x 5 = 10** **5 x 2 = 10**
Division with remainders	Children divide objects into groups to find out how many are left over/remaining.  They articulate what they are doing as they complete the calculation. ***14 divided into 3 groups equals 4 in each group with 2 left over.***	Children use a number line to jump forward in equal groups and then see how many more to find the remainder.  Children draw the whole as dots then group them.  	Children use their multiplication facts to divide with remainders. **37 ÷ 5** ***5 goes into 37 seven times, which is 35. The remainder is 2.*** **37 ÷ 5 = 7 r2**
Sharing using base 10	Children divide using a place value grid, regrouping when necessary.  	Children represent place value counters pictorially.  	

Short division	**615 ÷ 5** 1. Make 615 with place value counters.  2. How many groups of 5 hundreds can you make with 6 hundred counters? **(1 group, with 1 hundred remaining)** 3. Record above then exchange 1 hundred for 10 tens.  4. How many groups of 5 tens can you make with 11 ten counters? **(2 groups with 1 ten remaining)** 5. Record above then exchange 1 ten for 10 ones.  6. How many groups of 5 ones can you make with 15 ones? **(3 groups with none remaining)** 7. Record above to complete the calculation.  	**615 ÷ 5** 1. Write 615 (dividend) then draw the first line of the bus stop, saying ***divided,*** then the second line of the bus stop, saying ***by.*** Record the divisor (5). 2. Start on the left. How many times goes 5 go into 6? **(1 time, with a remainder of 1)** 3. Record above then regroup the remainder to the next column. $5\overline{)6^{1}15}$ (1) 4. How many times does 5 go into 11? **(2 times, with a remainder of 1)** 5. Record above then regroup the remainder. $5\overline{)6^{1}1^{1}5}$ (1 2) 6. How many times does 5 go into 15? **(3 times, with none remaining)** 7. Record to complete the calculation. $5\overline{)6^{1}1^{1}5}$ (1 2 3) If the calculation has a remainder, children learn to record this as a remainder, as a fraction, or to continue the calculation into decimal places.

Long division (dividing by a 1-digit number)

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
		
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.

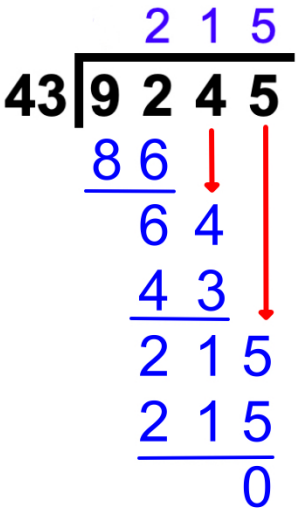
Divide.	Multiply & subtract.	Drop down the next digit.
		
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
		
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.

Long division (dividing by a 2-digit number)

Abstract only: Children follow the same process as above but before they start, they use their knowledge of times tables to write out multiples that will help them to solve the calculation.

9245 ÷ 43



Children learn to record remainders as decimals or as fractions, as appropriate to the calculation.

1	43
2	86
3	129
4	172
5	215
6	258
7	301
8	344
9	387
10	430

How to work out multiples of a number using related facts:	
x1	1 x 43 = 43
x2	Double 43 is 86
x3	43 + 86 = 129
x4	Double 86 is 172
x5	Half of 430 = 215
x6	Double 129 is 258
x7	3 x 7 = 21, 40 x 7 = 280 21 + 280 = 301
x8	Double 172 is 344
x9	430 – 43 = 387
x10	10 x 43 = 430

Find the multiples in this order:
1, 2, 4, 10, 5, 3, 6, 8, 9, 7

Encourage children not to work out 6, 7, 8 and 9 until they are sure they will need them.

Progression towards a formal written algorithm for division

Year 3	Year 4	Year 5	Year 6
Children use known and related facts to solve division calculations. They divide with remainders and use place value counters to divide – they work solely in the concrete/pictorial and do not use the abstract algorithm.	Children continue to practise the strategies learn in Year 3. If appropriate, they are introduced to short division, working in the concrete.	Children learn the concrete and abstract methods for short division (dividing by a 1-digit number), as detailed above.	Children learn to divide by a 2-digit number, using long division.